

Emily's Random Physics Problem

Question

A bullet with a mass of $m = 0.0021$ kg is initially moving at $v_0 = 507$ m/s. It imbeds itself in a large fixed piece of wood and travels $x = 0.76$ m into the wood before coming to rest. Assume that the acceleration is constant. What is the force exerted by the wood on the bullet?

Answer

The kinetic energy of the incoming bullet is

$$T = \frac{1}{2}mv_0^2$$

The work done by the block on the bullet is

$$W = -Fx$$

(It is negative because the force exerted is opposed to the direction of motion.)

If the bullet is to stop, its kinetic energy must be entirely depleted by the work done by the block. Therefore

$$\begin{aligned}T + W &= \frac{1}{2}mv_0^2 - Fx = 0 \\F &= \frac{mv_0^2}{2x} \\&\approx 355 \text{ N}\end{aligned}$$

Alternate solution

In order for the bullet to come to rest, it must decelerate over a certain amount of time.

$$\begin{aligned}0 &= v_0 - at \\ t &= \frac{v_0}{a}\end{aligned}$$

The acceleration is constant, so we can work to

$$\begin{aligned}\frac{d^2x}{dt^2} &= -a \\ \frac{dx}{dt} &= -at + v_0 \\ x &= -\frac{1}{2}at^2 + v_0t + x_0\end{aligned}$$

Plug in the time found before.

$$\begin{aligned}x &= -\frac{1}{2}a \left(\frac{v_0}{a}\right)^2 + v_0 \left(\frac{v_0}{a}\right) \\ &= -\frac{v_0^2}{2a} + \frac{v_0^2}{a} \\ &= \frac{v_0^2}{2a}\end{aligned}$$

Therefore

$$a = \frac{v_0^2}{2x}$$

and

$$\begin{aligned}F = ma &= \frac{mv_0^2}{2x} \\ &\approx 355 \text{ N}\end{aligned}$$

Alternate alternate solution

Force is the change in momentum over the change in time:

$$F = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t}$$

By substituting in a instead of t in the constant-acceleration equation we find Δt :

$$\begin{aligned}x &= -\frac{1}{2} \left(\frac{v_0}{t} \right) t^2 + v_0 t \\ &= -\frac{1}{2} v_0 t + v_0 t \\ &= \frac{1}{2} v_0 t \\ t &= \frac{2x}{v_0}\end{aligned}$$

The velocity changes from v_0 to 0 so Δv is just v_0 . Plug these in:

$$\begin{aligned}F &= mv_0 \frac{v_0}{2x} = \frac{mv_0^2}{2x} \\ &\approx 355 \text{ N}\end{aligned}$$